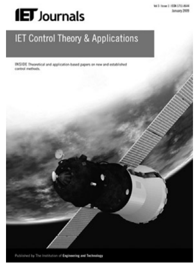
www.ietdl.org

Published in IET Control Theory and Applications
Received on 18th June 2013
Revised on 2nd October 2013
Accepted on 1st December 2013
doi: 10.1049/iet-cta.2013.0569



ISSN 1751-8644

New mixed-delay-dependent robust stability conditions for uncertain linear neutral systems

Yonggang Chen[1], *Shumin Fei*[2], *Zhou Gu*[2,3], *Yongmin Li*[2,4]

[1] *School of Mathematical Sciences, Henan Institute of Science and Technology, Xinxiang 453003, People's Republic of China*
[2] *Key Laboratory of Measurement and Control of CSE, Ministry of Education, School of Automation, Southeast University, Nanjing 210096, People's Republic of China*
[3] *College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, People's Republic of China*
[4] *School of Science, Huzhou Teachers College, Huzhou 313000, People's Republic of China*
E-mail: happycygzmd@tom.com

Abstract: In this study, mixed-delay-dependent robust stability problem is investigated for uncertain linear neutral systems with mixed delays. The existing stability conditions are obtained by employing the information of neutral delay and discrete delay independently, and are conservative to some extent. Different from most existing methods, this study attempts to introduce the interconnected information between neutral delay and discrete delay. Based on such an idea, the simple stability and robust stability conditions are firstly proposed by integral inequality method, then improved stability and robust stability conditions are obtained by incorporating delay-decomposition idea and augmented Lyapunov–Krasovskii functional. Theory analysis and examples show the benefits of the proposed techniques and conditions.

1 Introduction

Neutral systems have received much attention during the past two decades because of its wide applications in population ecology, distributed networks containing lossless transmission lines, heat exchangers etc. [1–25]. In particular, it is widely recognised that linear matrix inequality (LMI)-based stability conditions obtained by Lyapunov–Krasovskii (L-K) functional theory are more convenient for the controller design [3–25]. On the other hand, it is well known that delay-dependent stability criteria are generally less conservative than delay independent ones especially when the size of the delay is small. Therefore, over the past more than 10 years, various techniques have been developed to produce some delay-dependent stability and control design conditions, such as the bounding inequality for the cross term [5], descriptor model transformation [6–9], integral inequality methods [10, 11], free-weighting matrix techniques [10, 14–17], augmented L-K functionals [17, 19–22], delay-decomposition approaches [12, 23, 24] and complete L-K functional approach [13, 25]. Compared with the results in [13, 25], the conditions in [5–12, 14–24] are based on the simple L-K functionals, and remain conservative to some extent. However, the conditions in [5–12, 14–24] can be easily applied to controller synthesis.

For the delay-dependent conditions in [5–25], it should be pointed out that the neutral delay and discrete delay are assumed to be different in [5, 7–16] and equal in [6, 17–25]. For neutral systems with mixed delays, it is observed that the conditions in [5, 7–13] are neutral delay-independent and the results in [14–16] are neutral delay dependent. Compared with the neutral delay-independent conditions, the neutral delay-dependent conditions may be less conservative. However, it should be pointed out that the stability conditions in [14–16] employ the information of neutral delay and discrete delay independently, and the interconnected information between neutral delay and discrete delay is neglected. Therefore the mixed-delay-dependent stability conditions proposed in [14–16] are conservative to some extent, and some further investigations should be explored.

Motivated by the above discussions, this paper revisits robust stability problem for uncertain linear neutral systems with mixed delays. Different from the existing methods in [14–16], this paper introduces the interconnected information between neutral delay and discrete delay, which is well reflected by the new constructed L-K functionals. Combining with the integral inequality method, the simple stability and robust stability conditions are firstly proposed in terms of LMI. By integrating delay-decomposition approach and augmented L-K functional, the improved stability and robust stability conditions are further proposed. Compared with the mixed-delay-dependent conditions in [14–16], the stability conditions obtained in this paper are less conservative, which are well analysed by theory and illustrated by two numerical examples. In particular, it is worth mentioning that the mixed-delay-dependent conditions proposed in this paper will not introduce the additional conservatism for the case that neutral delay is equal to discrete delay.

Notation. Throughout this paper, the superscript 'T' stands for the transpose of a matrix. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and set of all $n \times n$ real matrices, respectively. A real symmetric matrix $P > 0\ (\geq 0)$ denotes P being a positive definite (positive semi-definite) matrix. I is used to denote an identity matrix with proper dimension. The symmetric terms in a symmetric matrix are denoted by $*$. $\|\cdot\|$ refers to the induced matrix 2-norm.

2 Problem formulation

Consider the following uncertain linear neutral system with mixed delays

$$\dot{x}(t) - (C + \Delta C(t))\dot{x}(t-\tau) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t-h) \quad (1)$$

$$x(t) = \phi(t), \quad \forall t \in [-\max\{\tau, h\}, 0] \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $h > 0$ and $\tau > 0$ denote the constant discrete delay and neutral delay, respectively. $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{n\times n}$ are known real constant matrices, $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ are unknown real matrices representing time-varying parameter uncertainties of system and are assumed to be of the following form

$$[\Delta A(t)\Delta B(t)\Delta C(t)] = DF(t)[E_a E_b E_c] \quad (3)$$

where D, E_a, E_b and E_c are known real constant matrices with appropriate dimensions, and $F(t)$ is an unknown continuous time-varying matrix function satisfying $F^{\mathrm{T}}(t)F(t) \leq I$.

Throughout this paper, it is assumed that the condition $\|C + \Delta C(t)\| \leq 1$ holds, which is necessary for guaranteeing the asymptotic stability of neutral system (1) and (2).

Lemma 1 [22]: For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, scalars a and b satisfying $a < b$, and vector function $\omega : [a, b] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$\text{(i)} \left(\int_a^b \omega(s)\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_a^b \omega(s)\,\mathrm{d}s\right) \leq (b-a)\int_a^b \omega^{\mathrm{T}}(s)M\omega(s)\,\mathrm{d}s$$

$$\text{(ii)} \left(\int_a^b \int_{t+\theta}^t \omega(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)^{\mathrm{T}} M \left(\int_a^b \int_{t+\theta}^t \omega(s)\,\mathrm{d}s\,\mathrm{d}\theta\right) \leq \frac{b^2-a^2}{2}\int_a^b \int_{t+\theta}^t \omega^{\mathrm{T}}(s)M\omega(s)\,\mathrm{d}s\,\mathrm{d}\theta$$

Lemma 2 [14]: Let $U, V, W$and M be real matrices of appropriate dimensions with M satisfying $M = M^{\mathrm{T}}$, then $M + UVW + W^{\mathrm{T}}V^{\mathrm{T}}U^{\mathrm{T}} < 0$ for all $V^{\mathrm{T}}V \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that $M + \varepsilon^{-1}UU^{\mathrm{T}} + \varepsilon W^{\mathrm{T}}W < 0$.

3 Simple stability conditions

To show the effectiveness of the proposed technique in this paper clearly, the simple stability and robust stability conditions will be presented in this section by integral inequality method. Firstly, we propose the asymptotic stability condition for the following nominal neutral system

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h) \quad (4)$$

Theorem 1: For given scalars τ and h, the nominal neutral system (4) is asymptotically stable, if there exist $n \times n$ matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $W > 0$, $R_1 > 0$, $R_2 > 0$ and $R_3 > 0$, such that the following LMI holds

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & A^{\mathrm{T}}Y \\ * & \Xi_{22} & \Xi_{23} & 0 & B^{\mathrm{T}}Y \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -W & C^{\mathrm{T}}Y \\ * & * & * & * & -Y \end{bmatrix} < 0 \quad (5)$$

where

$$\Xi_{11} = PA + A^{\mathrm{T}}P + Q_1 + Q_2 - R_1 - R_2, \Xi_{12} = PB + R_1$$
$$\Xi_{13} = -A^{\mathrm{T}}PC + R_2, \Xi_{22} = -Q_1 - R_1 + (\tau - h)Q_3 - R_3$$
$$\Xi_{23} = -B^{\mathrm{T}}PC + R_3, \Xi_{33} = -Q_2 - R_2 + (h - \tau)Q_3 - R_3$$
$$Y = h^2R_1 + \tau^2R_2 + (\tau - h)^2R_3 + W$$

Proof: Construct the following L-K functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (6)$$

where

$$V_1(t) = [x(t) - Cx(t-\tau)]^{\mathrm{T}}P[x(t) - Cx(t-\tau)]$$

$$V_2(t) = \int_{t-h}^t x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s + \int_{t-\tau}^t x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s + \int_{t-\tau}^t \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s + h\int_{-h}^0\int_{t+\theta}^t \dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \tau\int_{-\tau}^0\int_{t+\theta}^t \dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

$$V_3(t) = (\tau - h)\left[\int_{t-\tau}^{t-h} x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s \times \int_{-\tau}^{-h}\int_{t+\theta}^t \dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]$$

Differentiating $V_1(t)$, $V_2(t)$ and $V_3(t)$ along the trajectories of system (4) yield that

$$\dot{V}_1(t) = 2\left[x(t) - Cx(t-\tau)\right]^{\mathrm{T}}P[Ax(t) + Bx(t-h)] \quad (7)$$

$$\dot{V}_2(t) = x^{\mathrm{T}}(t)(Q_1 + Q_2)x(t) + \dot{x}^{\mathrm{T}}(t)(h^2R_1 + \tau^2R_2 + W)\dot{x}(t) - x^{\mathrm{T}}(t-h)Q_1x(t-h) - x^{\mathrm{T}}(t-\tau)Q_2x(t-\tau) - \dot{x}^{\mathrm{T}}(t-\tau)W\dot{x}(t-\tau) - h\int_{t-h}^t \dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s - \tau\int_{t-\tau}^t \dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s \quad (8)$$

$$\dot{V}_3(t) = (\tau - h)\left[x^{\mathrm{T}}(t-h)Q_3x(t-h) - x^{\mathrm{T}}(t-\tau)Q_3x(t-\tau)\right] + (\tau - h)^2\dot{x}^{\mathrm{T}}(t)R_3\dot{x}(t) - (\tau - h)\int_{t-\tau}^{t-h} \dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s \quad (9)$$

By the integral inequality (i) in Lemma 1, one can obtain that

$$-h\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\mathrm{d}s \le -\left(\int_{t-h}^{t}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}}R_1\left(\int_{t-h}^{t}\dot{x}(s)\mathrm{d}s\right)$$
$$= -[x(t)-x(t-h)]^{\mathrm{T}}R_1[x(t)-x(t-h)] \quad (10)$$

$$-\tau\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\mathrm{d}s \le -\left(\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}}R_2\left(\int_{t-\tau}^{t}\dot{x}(s)\mathrm{d}s\right)$$
$$= -[x(t)-x(t-\tau)]^{\mathrm{T}}R_2[x(t)-x(t-\tau)] \quad (11)$$

$$-(\tau-h)\int_{t-\tau}^{t-h}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s \le \left(\int_{t-\tau}^{t-h}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}}R_3\left(\int_{t-\tau}^{t-h}\dot{x}(s)\mathrm{d}s\right)$$
$$= -[x(t-h)-x(t-\tau)]^{\mathrm{T}}R_3[x(t-h)-x(t-\tau)] \quad (12)$$

Substituting the inequalities (10)–(12) into $\dot{V}(t)$, it can be concluded that

$$\dot{V}(t) \le \rho^{\mathrm{T}}(t)(\hat{\Xi}+\Phi^{\mathrm{T}}Y\Phi)\rho(t) \quad (13)$$

where

$$\rho(t)=\begin{bmatrix} x(t)\\ x(t-h)\\ x(t-\tau)\\ \dot{x}(t-\tau)\end{bmatrix},\quad \hat{\Xi}=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & 0\\ * & \Xi_{22} & \Xi_{23} & 0\\ * & * & \Xi_{33} & 0\\ * & * & * & -W\end{bmatrix},$$

$$\Phi=\begin{bmatrix}A^{\mathrm{T}}\\ B^{\mathrm{T}}\\ 0\\ C^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

and $Y, \Xi_{ij}, i,j=1,2,3$ have the same definitions as in (5). Using the well-known Schur complement, it is clear that $\Xi<0$ in (5) is equivalent to $\hat{\Xi}+\Phi^{\mathrm{T}}Y\Phi<0$, then it follows from (13) that $\dot{V}(t)<0$, which implies that system (4) is asymptotically stable. This completes the proof. □

For uncertain neutral system (1) with $\Delta C(t)=0$, based on the above Theorem 1, and combining with the Lemma 2, then the following robust stability condition can be easily obtained.

Theorem 2: For given scalars τ and h, the uncertain neutral system (1) is robustly asymptotically stable, if there exist $n\times n$ matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $W>0$, $R_1>0$, $R_2>0$, $R_3>0$ and a scalar $\mu>0$, such that the following LMI holds

$$\begin{bmatrix}\Xi & \tilde{D} & \mu\tilde{E}^{\mathrm{T}}\\ * & -\mu I & 0\\ * & * & -\mu I\end{bmatrix}<0 \quad (14)$$

where Ξ is defined in (5) and $\tilde{D}=\begin{bmatrix}D^{\mathrm{T}}P & 0 & -D^{\mathrm{T}}PC & 0 & D^{\mathrm{T}}Y\end{bmatrix}^{\mathrm{T}}$, $\tilde{E}=\begin{bmatrix}E_a & E_b & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$.

Remark 1: If the function $V_1(t)$ in (6) is chosen as $V_1(t)=x^{\mathrm{T}}(t)Px(t)$, then it is seen that the resulting LMI in Theorem 1 should be described by

$$\Lambda=\begin{bmatrix}\Xi_{11} & PB+R_1 & R_2 & PC & A^{\mathrm{T}}Y\\ * & \Xi_{22} & R_3 & 0 & B^{\mathrm{T}}Y\\ * & * & \Xi_{33} & 0 & 0\\ * & * & * & -W & C^{\mathrm{T}}Y\\ * & * & * & * & -Y\end{bmatrix}<0 \quad (15)$$

where $\Xi_{ii}, i=1,2,3, Y$ are defined in Theorem 1. Correspondingly, the robust stability condition for $\Delta C(t)\ne 0$ can be easily obtained by replacing the LMI in (14) with the following LMI

$$\begin{bmatrix}\Lambda & \breve{D} & \mu\breve{E}^{\mathrm{T}}\\ * & -\mu I & 0\\ * & * & -\mu I\end{bmatrix}<0 \quad (16)$$

where Λ is defined in (15) and $\breve{D}=\begin{bmatrix}D^{\mathrm{T}}P & 0 & 0 & 0 & D^{\mathrm{T}}Y\end{bmatrix}^{\mathrm{T}}$, $\breve{E}=\begin{bmatrix}E_a & E_b & 0 & E_c & 0\end{bmatrix}^{\mathrm{T}}$.

Remark 2: Compared with the L-K functional proposed in [14], $V_3(t)$ is further introduced in functional (6). It is observed that the functionals proposed in [14–16] employ the information of neutral delay τ and discrete delay h independently. However, the term $V_3(t)$ introduced in functional (6) reflects the relationship between neutral delay τ and discrete delay h, which results that Theorems 1 and 2 depend not only on τ and h, but also on $\tau-h$. In the following parts, we will show the importance of the functional $V_3(t)$ in reducing the possible conservatism.

It is well known that LMI (5) is equivalent to $\hat{\Xi}+\Phi^{\mathrm{T}}Y\Phi<0$, where $\hat{\Xi}$, Φ and Y have the same definitions as in (13). Noticing that $Y=Y_1+Y_2$, where $Y_1=h^2R_1+\tau^2R_2+W$ and $Y_2=(\tau-h)^2R_3$, then it is seen that LMI (5) is equivalent to $\hat{\Xi}+\Phi^{\mathrm{T}}Y_1\Phi+\Phi^{\mathrm{T}}Y_2\Phi<0$. When the term $(\tau-h)\int_{-\tau}^{-h}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$ is not introduced in (6), LMI (5) should be revised as

$$\Psi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & A^{\mathrm{T}}Y_1\\ * & \Xi_{22}+R_3 & \Xi_{23}-R_3 & 0 & B^{\mathrm{T}}Y_1\\ * & * & \Xi_{33}+R_3 & 0 & 0\\ * & * & * & -W & C^{\mathrm{T}}Y_1\\ * & * & * & * & -Y_1\end{bmatrix}<0 \quad (17)$$

where $\hat{\Xi}_{ij}, i,j=1,2,3$ have the same definitions as in (5). It is clear that LMI (17) is equivalent to $\hat{\Xi}+\Theta+\Phi^{\mathrm{T}}Y_1\Phi<0$ by Schur complement, where $\Theta=\mathrm{diag}\{0,\tilde{R},0\}$ and $\tilde{R}=\begin{bmatrix}R_3 & -R_3\\ -R_3 & R_3\end{bmatrix}$. Noticing the fact $\Theta\ge 0$, it follows from the inequality $\hat{\Xi}+\Theta+\Phi^{\mathrm{T}}Y_1\Phi<0$ that $\hat{\Xi}+\Phi^{\mathrm{T}}Y_1\Phi<0$. Then, it can be concluded that the inequality $\hat{\Xi}+\Phi^{\mathrm{T}}Y_1\Phi+\Phi^{\mathrm{T}}Y_2\Phi<0$ holds when $|\tau-h|$ is sufficiently small, which means that LMI (17) is a sufficient condition for LMI (5) for sufficiently small $|\tau-h|$. Reversely, one can not obtain LMI (17) by LMI (5) due to the fact $\Theta\ge 0$. Based on the above discussions, it can be seen that the conservatism of Theorem 1 can be reduced for sufficiently small $|\tau-h|$ when introducing the term $(\tau-h)\int_{-\tau}^{-h}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$ in $V_3(t)$.

doi: 10.1049/iet-cta.2013.0569

On the other hand, it is seen from LMI (5) that tuning variables $(\tau-h)Q_3$ and $(h-\tau)Q_3$ are introduced in the diagonal blocks Ξ_{22} and Ξ_{33}, respectively, because of the proposition of the term $(\tau-h)\int_{t-\tau}^{t-h}x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s$ in $V_3(t)$. It should be pointed out that the negative definition of the matrix Ξ in (5) is seriously effected by the diagonal blocks Ξ_{22} and Ξ_{33}. Therefore LMI (5) may become more slack for $\tau\neq h$ when tuning variables $(\tau-h)Q_3$ and $(h-\tau)Q_3$ are introduced in the matrix Ξ, which shows that the importance of the term $(\tau-h)\int_{t-\tau}^{t-h}x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s$ in $V_3(t)$.

Remark 3: The proposed L-K functional (6) is partly motivated by the functional in [26], where delay-dependent stability problem is investigated for linear systems with multiple discrete delays. However, different from the L-K functional proposed in [26], our constructed functional (6) includes the single integral term $(\tau-h)\int_{t-\tau}^{t-h}x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s$, which was not introduced in [26].

For the case that $\tau=h$, by the following functional and integral inequality (i) in Lemma 1

$$\begin{aligned}V_1(t)&=[x(t)-Cx(t-\tau)]^{\mathrm{T}}P[x(t)-Cx(t-\tau)]\\&\quad+\int_{t-\tau}^{t}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\\&\quad+\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s+\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\end{aligned}\tag{18}$$

the stability condition of nominal neutral system (4) can be described as follows.

Corollary 1: For given scalars τ and h, the nominal neutral system (4) is asymptotically stable, if there exist $n\times n$ matrices $P>0$, $Q>0$, $W>0$ and $R>0$, such that the following LMI holds

$$\Sigma=\begin{bmatrix}\Sigma_{11}&\Sigma_{12}&0&A^{\mathrm{T}}G\\ *&\Sigma_{22}&0&B^{\mathrm{T}}G\\ *&*&-W&C^{\mathrm{T}}G\\ *&*&*&-G\end{bmatrix}<0\tag{19}$$

where

$$\Sigma_{11}=PA+A^{\mathrm{T}}P+Q-R,\ \Sigma_{12}=PB-A^{\mathrm{T}}PC+R$$
$$\Sigma_{22}=-Q-R-C^{\mathrm{T}}PB-B^{\mathrm{T}}PC,\ G=\tau^2R+W$$

Remark 4: Let $R_1+R_2\triangleq R$, $Q_1+Q_2\triangleq Q$ and $h\triangleq\tau$, then it is easy to see that

$$\begin{bmatrix}I_{2n\times 2n}&\begin{pmatrix}0_{n\times n}\\ I_{n\times n}\end{pmatrix}&0_{2n\times 2n}\\ 0_{2n\times 2n}&0_{2n\times n}&I_{2n\times 2n}\end{bmatrix}\Xi\times\begin{bmatrix}I_{2n\times 2n}&\begin{pmatrix}0_{n\times n}\\ I_{n\times n}\end{pmatrix}&0_{2n\times 2n}\\ 0_{2n\times 2n}&0_{2n\times n}&I_{2n\times 2n}\end{bmatrix}^{\mathrm{T}}=\Sigma\tag{20}$$

which means that Theorem 1 is equivalent to Corollary 5 for the case that $\tau=h$.

Remark 5: When the functional $V_3(t)$ is abandoned in (6), the corresponding stability conditions can be easily obtained by setting $Q_3=R_3=0$ in Theorems 1 and 2, which are referred to as **Corollaries 1 and 2**, respectively. In the case that the term $(\tau-h)\int_{t-\tau}^{t-h}x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s$ is not introduced in $V_3(t)$, by setting $Q_3=0$ in Theorems 1 and 2, one can also obtain the corresponding stability conditions, which are referred to as **Corollaries 3 and 4**, respectively.

4 Improved stability conditions

In this section, the improved stability and robust stability conditions will be established by incorporating the idea of delay-decomposition and augmented L-K functional.

Theorem 3: For given scalars τ, h and N, the nominal neutral system (4) is asymptotically stable, if there exist $3n\times 3n$ matrix $P=\begin{bmatrix}P_{11}&P_{12}&P_{13}\\ P_{12}^{\mathrm{T}}&P_{22}&P_{23}\\ P_{13}^{\mathrm{T}}&P_{23}^{\mathrm{T}}&P_{33}\end{bmatrix}>0$, $2n\times 2n$ matrices $W=\begin{bmatrix}W_{11}&W_{12}\\ W_{12}^{\mathrm{T}}&W_{22}\end{bmatrix}>0$, $Z=\begin{bmatrix}Z_{11}&Z_{12}\\ Z_{12}^{\mathrm{T}}&Z_{22}\end{bmatrix}>0$ and $n\times n$ matrices $S>0$, $Q_i>0$, $R_i>0$, $U_i>0$, $H_i>0$, $i=1,2,\ldots,N$, T_1,T_2,T_3, such that the following LMI holds (see (21)) where

$$\begin{aligned}\Omega_1^1&=T_1A+A^{\mathrm{T}}T_1^{\mathrm{T}}+P_{13}+P_{13}^{\mathrm{T}}+W_{11}+\tau^2Z_{11}\\&\quad+Q_1-Z_{22}-\tau^2S-R_1\\ \Omega_{N+2}^1&=-P_{13}+P_{23}^{\mathrm{T}}+Z_{22},\quad \Omega_{N+3}^1=P_{33}-Z_{12}^{\mathrm{T}}+\tau S\\ \Omega_{N+4}^1&=P_{11}+W_{12}+\tau^2Z_{12}-T_1+A^{\mathrm{T}}T_2^{\mathrm{T}},\end{aligned}$$

$$\Omega=\begin{bmatrix}\Omega_1^1&R_1&0&\cdots&0&T_1B&\Omega_{N+2}^1&\Omega_{N+3}^1&\Omega_{N+4}^1&\Omega_{N+5}^1\\ *&\Omega_2^2&R_2&\cdots&0&0&H_1&0&0&0\\ *&*&\Omega_3^3&\cdots&0&0&H_2&0&0&0\\ \vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\ *&*&*&*&\Omega_N^N&R_N&H_{N-1}&0&0&0\\ *&*&*&*&*&\Omega_{N+1}^{N+1}&H_N&0&B^{\mathrm{T}}T_2^{\mathrm{T}}&B^{\mathrm{T}}T_3^{\mathrm{T}}\\ *&*&*&*&*&*&\Omega_{N+2}^{N+2}&\Omega_{N+3}^{N+2}&P_{12}^{\mathrm{T}}&\Omega_{N+5}^{N+2}\\ *&*&*&*&*&*&*&\Omega_{N+3}^{N+3}&P_{13}^{\mathrm{T}}&P_{23}^{\mathrm{T}}\\ *&*&*&*&*&*&*&*&\Omega_{N+4}^{N+4}&\Omega_{N+5}^{N+4}\\ *&*&*&*&*&*&*&*&*&\Omega_{N+5}^{N+5}\end{bmatrix}<0\tag{21}$$

$$\Omega^1_{N+5} = P_{12} + T_1C + A^{\mathrm{T}}T_3^{\mathrm{T}}$$
$$\Omega^{i+1}_{i+1} = -Q_i + Q_{i+1} - R_i - R_{i+1} + (\tau - ih/N)U_i - H_i,\quad i = 1, 2, \ldots, N-1$$
$$\Omega^{N+1}_{N+1} = -Q_N - R_N + (\tau - h)U_N - H_N$$
$$\Omega^{N+2}_{N+2} = -W_{11} - P_{23} - P_{23}^{\mathrm{T}} - Z_{22} - \Sigma_{i=1}^N[(\tau - ih/N)U_i + H_i]$$
$$\Omega^{N+2}_{N+3} = -P_{33} + Z_{12}^{\mathrm{T}},\quad \Omega^{N+2}_{N+5} = P_{22} - W_{12},$$
$$\Omega^{N+3}_{N+3} = -Z_{11} - S$$
$$\Omega^{N+4}_{N+4} = W_{22} + \tau^2 Z_{22} + (\tau^4/4)S + \Sigma_{i=1}^N[(h/N)^2R_i + (\tau - ih/N)^2H_i] - T_2 - T_2^{\mathrm{T}}$$
$$\Omega^{N+4}_{N+5} = T_2C - T_3^{\mathrm{T}},\quad \Omega^{N+5}_{N+5} = -W_{22} + T_3C + C^{\mathrm{T}}T_3^{\mathrm{T}}$$

Proof: Construct the following L-K functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{22}$$

where

$$V_1(t) = \xi^{\mathrm{T}}(t)P\xi(t) + \int_{t-\tau}^{t} \eta^{\mathrm{T}}(s)W\eta(s)\mathrm{d}s + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \eta^{\mathrm{T}}(s)Z\eta x(s)\mathrm{d}s\mathrm{d}\theta + (\tau^2/2)\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s)S\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta$$

$$V_2(t) = \sum_{i=1}^{N}\left[\int_{t-ih/N}^{t-(i-1)h/N} x^{\mathrm{T}}(s)Q_ix(s)\mathrm{d}s + (h/N)\int_{-ih/N}^{-(i-1)h/N}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)R_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]$$

$$V_3(t) = \sum_{i=1}^{N}(\tau - ih/N)\left[\int_{t-\tau}^{t-ih/N} x^{\mathrm{T}}(s)U_ix(s)\mathrm{d}s + \int_{-\tau}^{-ih/N}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)H_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right]$$

and $\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)\ \ x(t-\tau)(\int_{t-\tau}^{t} x(s)\,\mathrm{d}s)^{\mathrm{T}}]$, $\eta^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)\ \ \dot{x}^{\mathrm{T}}(t)]$. Differentiating $V_1(t)$, $V_2(t)$ and $V_3(t)$ along the trajectories of nominal system (4), then one obtain that (see (23–25))

By the integral inequalities in Lemma 1, one can obtain that

$$-\tau\int_{t-\tau}^{t}\eta^{\mathrm{T}}(s)Z\eta x(s)\mathrm{d}s \le -\left(\int_{t-\tau}^{t}\eta(s)\mathrm{d}s\right)^{\mathrm{T}} Z\left(\int_{t-\tau}^{t}\eta(s)\mathrm{d}s\right) = -\begin{bmatrix}\int_{t-\tau}^{t}x(s)\mathrm{d}s\\ x(t)-x(t-\tau)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{12}^{\mathrm{T}} & Z_{22}\end{bmatrix}\begin{bmatrix}\int_{t-\tau}^{t}x(s)\mathrm{d}s\\ x(t)-x(t-\tau)\end{bmatrix} \tag{26}$$

$$-(\tau^2/2)\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)S\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right)^{\mathrm{T}} S\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right) = -\left[\tau x(t) - \int_{t-\tau}^{t}x(s)\mathrm{d}s\right]^{\mathrm{T}} S\left[\tau x(t) - \int_{t-\tau}^{t}x(s)\mathrm{d}s\right] \tag{27}$$

$$-(h/N)\int_{t-ih/N}^{t-(i-1)h/N}\dot{x}^{\mathrm{T}}(s)R_i\dot{x}(s)\mathrm{d}s \le -\left(\int_{t-ih/N}^{t-(i-1)h/N}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}} R_i\left(\int_{t-ih/N}^{t-(i-1)h/N}\dot{x}(s)\mathrm{d}s\right) = -[x(t-(i-1)h/N) - x(t-ih/N)]^{\mathrm{T}} \times R_i[x(t-(i-1)h/N) - x(t-ih/N)] \tag{28}$$

$$\begin{aligned}\dot{V}_1(t) = {} & 2\left[x^{\mathrm{T}}(t)P_{11} + x^{\mathrm{T}}(t-\tau)P_{12}^{\mathrm{T}} + \int_{t-\tau}^{t}x^{\mathrm{T}}(s)\mathrm{d}sP_{13}^{\mathrm{T}}\right]\dot{x}(t) + 2\left[x^{\mathrm{T}}(t)P_{12} + x^{\mathrm{T}}(t-\tau)P_{22} + \int_{t-\tau}^{t}x^{\mathrm{T}}(s)\mathrm{d}sP_{23}^{\mathrm{T}}\right]\dot{x}(t-\tau)\\ & + 2\left[x^{\mathrm{T}}(t)P_{13} + x^{\mathrm{T}}(t-\tau)P_{23} + \int_{t-\tau}^{t}x^{\mathrm{T}}(s)\mathrm{d}sP_{33}\right][x(t) - x(t-\tau)]\\ & + \eta^{\mathrm{T}}(t)(W + \tau^2 Z)\eta(t) + (\tau^4/4)\dot{x}^{\mathrm{T}}(t)S\dot{x}(t) - \eta^{\mathrm{T}}(t-\tau)W\eta(t-\tau)\\ & - \tau\int_{t-\tau}^{t}\eta^{\mathrm{T}}(s)Z\eta x(s)\mathrm{d}s - (\tau^2/2)\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)S\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{V}_2(t) = \sum_{i=1}^{N}\Big[& x^{\mathrm{T}}(t-(i-1)h/N)Q_ix(t-(i-1)h/N) - x^{\mathrm{T}}(t-ih/N)Q_ix(t-ih/N)\\ & + (h/N)^2\dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t) - (h/N)\int_{t-ih/N}^{t-(i-1)h/N}\dot{x}^{\mathrm{T}}(s)R_i\dot{x}(s)\mathrm{d}s\Big]\end{aligned} \tag{24}$$

$$\begin{aligned}\dot{V}_3(t) = {} & \sum_{i=1}^{N}(\tau - ih/N)\left[x^{\mathrm{T}}(t-ih/N)U_ix(t-ih/N) - x^{\mathrm{T}}(t-\tau)U_ix(t-\tau) - \int_{t-\tau}^{t-ih/N}\dot{x}^{\mathrm{T}}(s)H_i\dot{x}(s)\mathrm{d}s\right]\\ & + \sum_{i=1}^{N}(\tau - ih/N)^2\dot{x}^{\mathrm{T}}(t)H_i\dot{x}(t)\end{aligned} \tag{25}$$

doi: 10.1049/iet-cta.2013.0569

$$-(\tau - ih/N)\sum_{i=1}^{N}\int_{t-\tau}^{t-ih/N}\dot{x}^{\mathrm{T}}(s)H_i\dot{x}(s)\mathrm{d}s$$
$$\le -\left(\int_{t-\tau}^{t-ih/N}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}} H_i\left(\int_{t-\tau}^{t-ih/N}\dot{x}(s)\mathrm{d}s\right)$$
$$= -[x(t-ih/N)-x(t-\tau)]^{\mathrm{T}}H_i[x(t-ih/N)-x(t-\tau)] \tag{29}$$

For any matrices T_1, T_2 and T_3, it follows from the system (4) that

$$2[x^{\mathrm{T}}(t)T_1 + \dot{x}^{\mathrm{T}}(t)T_2 + \dot{x}^{\mathrm{T}}(t-\tau)T_3] \times [Ax(t) + Bx(t-h) - \dot{x}(t) + C\dot{x}(t-\tau)] = 0 \tag{30}$$

Adding the left-hand side of (30) to $\dot{V}(t)$, and combining with (23)–(25) and the inequalities (26)–(29), then it is easy to see that $\dot{V}(t)$ can be enlarged as

$$\dot{V}(t) \le \zeta^{\mathrm{T}}(t)\Omega\zeta(t) \tag{31}$$

where Ω is defined in (21), and $\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-h/N)\ \ x^{\mathrm{T}}(t-2h/N)\cdots\ \ x^{\mathrm{T}}(t-h)\ \ x^{\mathrm{T}}(t-\tau)\ \ (\int_{t-\tau}^{t}x(s)\mathrm{d}s)^{\mathrm{T}}\ \ \dot{x}^{\mathrm{T}}(t)\ \ \dot{x}^{\mathrm{T}}(t-\tau)]$. Note that $\Omega < 0$ in (21), it is clear from (31) that $\dot{V}(t) < 0$, which implies that nominal system (4) is asymptotically stable. This completes the proof. □

Remark 6: The constructed L-K functional (22) includes three parts, where augmented functional $V_1(t)$ is utilised to cope with neutral delay τ, $V_2(t)$ is based on the delay-decomposition idea proposed in [12] and is utilised to cope with discrete delay h, and $V_3(t)$ is utilised to reflect the relationship between neutral delay τ and decomposed discrete delay ih/N. If the idea of delay-decomposition is further incorporated to deal with neutral delay τ, it is expected that some more effective conditions can be obtained. Compared with the L-K functional (6), it is seen that the techniques of augmented functional and delay-decomposition are incorporated when constructing functional (22). Therefore it is possible that Theorem 3 is less conservative than Theorem 1.

Using Theorem 3 and Lemma 2, one can obtain the following robust stability condition.

Theorem 4: For given scalars τ, h and N, the uncertain neutral system (1) is robustly asymptotically stable, if there exist $3n\times 3n$ matrix $P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12}^{\mathrm{T}} & P_{22} & P_{23} \\ P_{13}^{\mathrm{T}} & P_{23}^{\mathrm{T}} & P_{33}\end{bmatrix} > 0$, $2n\times 2n$ matrices $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^{\mathrm{T}} & W_{22}\end{bmatrix} > 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22}\end{bmatrix} > 0$, $n\times n$ matrices $S > 0$, $Q_i > 0$, $R_i > 0$, $U_i > 0$, $H_i > 0$, $i = 1, 2, \ldots, N$, T_1, T_2, T_3, and a scalar $\mu > 0$, such that the following LMI holds

$$\begin{bmatrix} \Omega & \bar{D} & \mu\bar{E}^{\mathrm{T}} \\ * & -\mu I & 0 \\ * & * & -\mu I\end{bmatrix} < 0 \tag{32}$$

where Ω is defined in (21) and

$$\bar{D} = \begin{bmatrix} D^{\mathrm{T}}T_1^{\mathrm{T}} & \underbrace{\overset{N+2}{0\cdots 0}} & D^{\mathrm{T}}T_2^{\mathrm{T}} & D^{\mathrm{T}}T_3^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$\bar{E} = \begin{bmatrix} E_a & \underbrace{\overset{N-1}{0\cdots 0}} & E_b & 0 & 0 & 0 & E_c\end{bmatrix}^{\mathrm{T}}$$

5 Numerical examples

Example 1: Consider the nominal neutral system (4) with the following parameters

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9\end{bmatrix},\quad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1\end{bmatrix},$$
$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1\end{bmatrix}$$

For this example, the maximum admissible delay bounds are computed by the conditions in this paper and the results in [11, 12, 14–16, 23–25], which are listed in Tables 1 and 2. It is clear from Table 1 that the existing mixed mixed-delay-dependent conditions in [14–16] are conservative because of the neglect of the relationship between neutral delay τ and

Table 1 Maximum admissible upper bounds of h for different τ (Example 1)

τ	0.1	0.3	0.5	0.7	0.9	1.0	1.2
He *et al.* [14], Corollary 1	1.7100	1.6883	1.6718	1.6624	1.6563	1.6543	1.6527
Liu *et al.* [15]	1.7844	1.7669	1.7495	1.7338	1.7226	1.7201	1.7191
Qian *et al.* [16]	1.8307	1.8038	1.7755	1.7484	1.7272	1.7213	1.7202
Theorem 1	1.7808	1.8855	1.9718	2.0387	2.0886	2.1052	2.1145
Corollary 3	1.7236	1.7286	1.7352	1.7429	1.7540	1.7610	1.7733
Theorem 3 ($N = 1$)	1.8413	1.9778	2.0954	2.1877	2.2449	2.2611	2.2711
Theorem 3 ($N = 3$)	2.1845	2.2521	2.2973	2.3204	2.3301	2.3331	2.3313
Theorem 3 ($N = 5$)	2.2137	2.2774	2.3210	2.3474	2.3568	2.3588	2.3569

Table 2 Maximum admissible upper bounds for $\tau = h$ (Example 1)

Han [12]	Han *et al.* [14]	Liu *et al.* [15]	Qian *et al.* [16]	Han [11], Corollary 5
2.2036	1.6527	1.7191	1.7197	1.7856
Theorem 1	Balasubramaniam *et al.* [23] ($N = 5$)	Kwon *et al.* [24]	Han *et al.* [25] ($N = 4$)	Theorem 3 ($N = 5$)
1.7856	2.1980	2.1633	2.2254	2.2069

Table 3 Maximum admissible upper bounds of $h = \tau$ for different c (Example 2)

c	0.00	0.05	0.10	0.15	0.20	0.25	0.30
He *et al.* [14]	2.39	2.05	1.75	1.49	1.27	1.08	0.91
Liu *et al.* [15]	2.39	2.13	1.89	1.67	1.48	1.30	1.15
Qian *et al.* [16]	2.44	2.17	1.93	1.72	1.52	1.35	1.19
Theorem 2, Han [11]	2.39	2.12	1.86	1.63	1.42	1.23	1.06
Theorem 4 ($N = 1$)	2.75	2.64	2.53	2.40	2.26	2.11	1.95
Theorem 4 ($N = 3$)	3.05	2.91	2.76	2.60	2.44	2.26	2.08
Theorem 4 ($N = 5$)	3.10	2.96	2.80	2.64	2.47	2.29	2.11

discrete delay h, and Theorem 3 is less conservative than Theorem 1 because of the introductions of the techniques of augmented L-K functional and delay-decomposition. In addition, it is seen from Table 1 that Theorem 1 can provide larger delay bounds than Corollary 3, which shows the importance of the single integral $(\tau - h)\int_{t-\tau}^{t-h} x^{\mathrm{T}}(s)Q_3x(s)\mathrm{d}s$ introduced in L-K functional (6). From Table 2, It is seen that the Theorem 1 in this paper provides the same delay bound $h = \tau = 1.7856$ as the Proposition 3 in [11] and Corollary 5 in this paper, and a larger delay bound than the mixed mixed-delay-dependent conditions in [14–16], which verifies the Remark 5 and shows the effectiveness of the proposed technique in this paper. Also, Table 2 shows that Theorem 3 in this paper provides a larger delay bound than the conditions in [11, 12, 14–16, 23, 24] obtained by the simple L-K functionals. Compared with the condition in [26], a slightly smaller delay bound is obtained by Theorem 3 in this paper. Recalling that the neural delay and discrete delay are assumed to be equal in [25], and the condition in [25] is obtained by complete L-K functional approach, it is clear that the Theorem 3 proposed in this paper remains interesting, since Theorem 3 can be applicable to neutral systems with mixed delays and is convenient for the controller synthesis and filter design.

Example 2: Consider the uncertain neutral system (1) with the following parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$
$$0 \le c < 1, \quad D = I, \quad E_a = E_b = 0.2I, \quad E_c = 0$$

For this example, Table 3 lists the maximum admissible delay bounds for different c by Theorems 2 and 4 proposed in this paper and the conditions in [11, 14–16]. It can be seen from Table 3 that Theorem 2 recovers the delay bounds obtained by the Proposition 3 in [11], and Theorem 4 provides the larger delay bounds than the mixed-delay-dependent conditions in [14–16].

For system without uncertainties, Table 4 lists the maximum admissible delay bounds for different c by Theorem 3 obtained in this paper and the conditions in [22–25]. Table 4 shows that Theorem 3 in this paper provides larger delay bounds than the condition in [22] and similar delay bounds as the conditions in [23–25], which shows the Theorem 3 in this paper remains effective for the systems with equal delays. Noticing that the conditions in [22–25] are obtained under the assumption of equal delays, clearly, the proposed Theorem 3 in this paper is more interesting.

6 Conclusion

In this paper, the simple and improved mixed-delay-dependent stability and robust stability conditions are proposed for uncertain linear neutral systems with mixed delays. Compared with some existing results, the obtained conditions in this paper are based on the simple L-K functional, and can reduce the possible conservatism because of the introduction of the interconnected information between neutral delay and discrete delay. Theory analysis and numerical examples show the benefits of the proposed conditions and techniques. The proposed techniques in this paper can be extended to neutral systems with mixed time-varying delays and multiple delays. In addition, the controller synthesis and filter design can be easily performed by the conditions in this paper.

7 Acknowledgments

This work was supported by the National Natural Science Foundations of PR China under Grants nos. 61304061, 61273119 and 61174076. This work was also partly supported by the Scientific Research Program of Education Department of Zhejiang Province under Grant no. Y201122032.

8 References

1 Park, Ju H.: 'Simple criterion for asymptotic stability of interval neutral delay-differential systems', *Appl. Math. Lett.*, 2003, **16**, (7), pp. 1063–1068
2 Cao, D.Q., He, P.: 'Stability criteria of linear neutral systems with a single delay', *Appl. Math. Comput.*, 2004, **148**, (1), pp. 135C143
3 Xu, S., Lam, J., Yang, C.: 'H_∞ and positive-real control for linear neutral delay systems', *IEEE Trans. Autom. Control*, 2001, **46**, (8), pp. 1321–1326

Table 4 Maximum admissible upper bounds of $h = \tau$ for different c (Example 2)

c	0.0	0.1	0.3	0.5	0.7	0.9
Sun *et al.* [22]	5.30	5.21	4.85	4.20	3.19	1.49
Balasubramaniam *et al.* [23]($N = 5$)	6.10	5.97	5.49	4.69	3.50	1.55
Kwon *et al.* [24]	5.94	5.82	5.36	4.60	3.42	1.54
Han *et al.* [25]($N = 3$)	6.17	6.03	5.54	4.73	3.50	1.57
Theorem 3 ($N = 1$)	5.30	5.21	4.85	4.20	3.19	1.49
Theorem 3 ($N = 3$)	5.96	5.84	5.38	4.61	3.43	1.55
Theorem 3 ($N = 5$)	6.09	5.96	5.48	4.69	3.48	1.56

doi: 10.1049/iet-cta.2013.0569

4 Li, Y., Xu, S., Zhang, B., Chu, Y.: 'Robust stabilization and H_∞ control for uncertain fuzzy neutral systems with mixed time delays', *Fuzzy Sets Syst.*, 2008, **159**, (20), pp. 2730–2748
5 Park, Ju H.: 'Stability criterion for neutral differential systems with mixed multiple time-varying delay arguments', *Math. Comput. Simulation*, 2002, **59**, (5), pp. 401–412
6 Fridman, E.: 'New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems', *Syst. Control Lett.*, 2001, **43**, (4), pp. 309–319
7 Fridman, E., Shaked, U.: 'A descriptor system approach to H_∞ control of linear time-delay systems', *IEEE Trans. Autom. Control*, 2002, **47**, (2), pp. 253–270
8 Fridman, E., Shaked, U.: 'Delay-dependent stability and H_∞ control: constant and time-varying delays', *Int. J. Control*, 2003, **76**, (1), pp. 48–60
9 Chen, W.-H., Zheng, W.X.: 'Delay-dependent robust stabilization for uncertain neutral systems with distributed delays', *Automatica*, 2007, **43**, (1), pp. 95–104
10 Kwon, O.M., Park, Ju H., Lee, S.M.: 'On stability criteria for uncertain delay-differential systems of neutral type with time-varying delays', *Appl. Math. Comput.*, 2008, **197**, (2), pp. 864–873
11 Han, Q.-L.: 'New delay-dependent stability criterion for linear neutral systems with norm-bounded uncertainties in all system matrices', *Int. J. Syst. Sci.*, 2005, **36**, (8), pp. 469–475
12 Han, Q.-L.: 'Improved stability criteria and controller design for linear neutral systems', *Automatica*, 2009, **45**, (8), pp. 1948–1952
13 Han, Q.-L.: 'On stability of linear neutral systems with mixed time-delays: A discretized Lyapunov functional approach', *Automatica*, 2005, **41**, (7), pp. 1209–1218
14 He, Y., Wu, M., She, J.-H., Liu, G.-P.: 'Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays', *Syst. Control Lett.*, 2004, **51**, (1), pp. 57–65
15 Liu, X.-G., Wu, M., Martin, R., Tang, M.-L.: 'Stability analysis for neutral systems with mixed delays', *J. Comput. Appl. Math.*, 2007, **202**, (2), pp. 478–497
16 Qian, W., Liu, J., Sun, Y., Fei, S.: 'A less conservative robust stability criteria for uncertain neutral systems with mixed delays', *Math. Comput. Simulation*, 2010, **80**, (5), pp. 1007–1017
17 Wu, M., He, Y., She, J.-H.: 'New delay-dependent stability criteria and stabilizing method for neutral systems', *IEEE Trans. Autom. Control*, 2004, **49**, (12), pp. 2266–2271
18 He, Y., Wang, Q.-G, Lin, C., Wu, M.: 'Augmented Lyapunov functional and delay-dependent stability criteria for neutral systems', *Int. J. Robust Nonlinear Control*, 2005, **15**, (18), pp. 923–933
19 Parlakci, M.N.A.: 'Robust stability of uncertain neutral systems: a novel augmented Lyapunov functional approach', *IET Control Theory Appl.*, 2007, **1**, (3), pp. 802–809
20 Parlakci, M.N.A.: 'Extensively augmented Lyapunov functional for the stability of neutral time-delay systems', *IET Control Theory Appl.*, 2008, **2**, (5), pp. 431–436
21 Zhang, X.-M., Han, Q.-L.: 'Stability analysis and H_∞ filtering for delay differential systems of neutral type', *IET Control Theory Appl.*, 2007, **1**, (3), pp. 749–755
22 Sun, J., Liu, G.P., Chen, J.: 'Delay-dependent stability and stabilization of neutral time-delay systems', *Int. J. Robust Nonlinear Control*, 2009, **19**, (1), pp. 1364–1375
23 Balasubramaniam, P., Krishnasamy, R., Rakkiyappan, R.: 'Delay-dependent stability of neutral systems with time-varying delays using delay-decomposition approach', *Appl. Math. Model.*, 2012, **36**, (5), pp. 2253–2261
24 Kwon, O.M., Park, M.J., Park, Ju H., Lee, S.M., Cha, E.J.: 'New delay-partitioning approaches to stability criteria for uncertain neutral systems with time-varying delays', *J. Franklin Inst.*, 2012, **349**, (9), pp. 2799–2823
25 Han, Q.-L., Yu, X., Gu, K.: 'On computing the maximum time-delay bound for stability of linear neutral systems', *IEEE Trans. Autom. Control*, 2004, **49**, (12), pp. 2281–2286
26 He, Y., Wu, M., She, J.-H.: 'Delay-dependent stability criteria for linear systems with multiple time delays', *IEE Proc.-Control Theory Appl.*, 2006, **153**, (4), pp. 447–452